Mark Scheme (Results)

Summer 2022

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

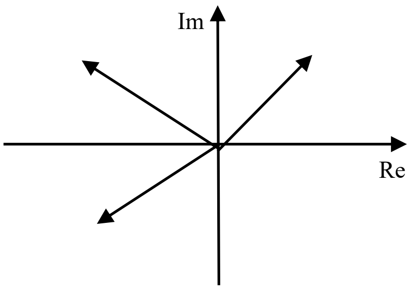
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1(a)	$z_1 = 3+3\mathrm{i} \qquad z_2 = p+q\mathrm{i} \qquad p,q \in \mathbb{R}$		
	$\lvert z_1\rvert = \sqrt{3^2+3^2}$ $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert \Rightarrow \lvert z_2\rvert\sqrt{18} = 15\sqrt{2} \Rightarrow \lvert z_2\rvert = \ldots$	Attempts $\lvert z_1\rvert$ using Pythagoras and uses $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ to find $\lvert z_2\rvert$	M1
	$\lvert z_2\rvert = 5$	Cao	A1
			(2)
ALT	$\lvert z_1 z_2\rvert = 15\sqrt{2}$ $\lvert(3p-3q)+\mathrm{i}(3p+3q)\rvert = 15\sqrt{2}$ $\sqrt{18p^2+18q^2} = 15\sqrt{2}$ $p^2+q^2 = 25$ $\lvert z_2\rvert = \sqrt{p^2+q^2} = 5$	Uses $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ to reach $p^2+q^2 = \ldots$	M1 A1 **(2)**
(b)	$\lvert z_2\rvert = 5 \Rightarrow p^2+q^2 = 25$ $\Rightarrow (-4)^2 + q^2 = 25 \Rightarrow q = \ldots$	Uses $p^2+q^2 = \text{"}5\text{"}^2$ with $p = \pm 4$ leading to a value for q.	M1
	$q = \pm 3$	Both values. Must be clear $p = 4$ has not been used	A1
			(2)
(c)	 Points to be in the correct quadrants and either with correct numbers on the axes or labelled correctly	3 + 3i plotted correctly and labelled Vectors/ lines not needed; point(s) alone are sufficient	B1
		A conjugate pair plotted correctly following through their q.	B1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2	$f(x)=10-2x-\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x^3} \qquad x>0$		
(a)	$f(0.4)=-7.21...,\ f(0.5)=0.292...$	Attempts both f(0.4) and f(0.5)	M1
	Sign change (positive, negative) and $f(x)$ is **continuous** therefore (a root) α is between $x=0.4$ and $x=0.5$	Both f(0.4) = awrt −7 and f(0.5) = awrt 0.3, sign change and conclusion. **Must mention continuity**. Can have $f(0.4)\times f(0.5)<0$ instead of "sign change"	A1
			(2)
(b)	$f'(x)=-2+\dfrac{1}{4}x^{-\frac{3}{2}}+3x^{-4}$	$x^n \to x^{n-1}$ in at least 1 term other than 10	M1
		2 of the 3 terms shown correct	A1
		All correct	A1
			(3)
(c)	$x_1=0.5-\dfrac{f(0.5)}{f'(0.5)}=0.5-\dfrac{0.29289321...}{46.70710678...}$	Correct application of Newton-Raphson	M1
	$=0.494$	Correct value 3dp. A correct derivative must have been used	A1
			(2)
(d)	$\dfrac{4.9-\beta}{\lvert f(4.9)\rvert}=\dfrac{\beta-4.8}{f(4.8)}\Rightarrow\beta=...$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 1	$\beta=\dfrac{a\lvert f(b)\rvert+b\lvert f(a)\rvert}{\lvert f(a)\rvert+\lvert f(b)\rvert}$ $\beta=\dfrac{4.8\times0.0344+4.9\times0.1627}{0.0344+0.1627}=...$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 2	Gradient $=\dfrac{-0.0344-0.1627}{4.9-4.8}=-1.971$ Equation of line: $y-0.1627=-1.971(x-4.8)$ or $y=-1.971+9.6235$ Substitute $y=0$ $x=...$	Complete method for line equation followed by substitution to obtain a value for x	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3(a)	$\mathbf{M}^{-1}=\frac{1}{5k-3k}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$	Attempts $\mathbf{M}^{-1}=\frac{1}{\det\mathbf{M}}\times\operatorname{adj}(\mathbf{M})$ Either part correct but $\operatorname{adj}(\mathbf{M})=\mathbf{M}$ scores M0	M1
	$=\frac{1}{2k}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$ or $\begin{pmatrix}\frac{5}{2k} & -\frac{1}{2}\\ \frac{-3}{2k} & \frac{1}{2}\end{pmatrix}$	Correct matrix $2k$ must be seen for this mark	A1
			(2)
(b)	$(\mathbf{MN})^{-1}=\mathbf{N}^{-1}\mathbf{M}^{-1}=\frac{1}{2k}\begin{pmatrix}k & k\\ 4 & -1\end{pmatrix}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$	Applies $(\mathbf{MN})^{-1}=\mathbf{N}^{-1}\mathbf{M}^{-1}$	M1
	$=\frac{1}{2k}\begin{pmatrix}2k & 0\\ 23 & -5k\end{pmatrix}$ or e.g. $\begin{pmatrix}1 & 0\\ \frac{23}{2k} & \frac{-5}{2}\end{pmatrix}$	Correct matrix	A1
			(2)
ALT (b)	Find **N** (ie inverse of $\mathbf{N}^{-1}$) Find **MN** $=-\frac{1}{5k}\begin{pmatrix}-5k & 0\\ -23 & 2k\end{pmatrix}$ Find $(\mathbf{MN})^{-1}$	Complete method needed	M1
	$=\frac{1}{2k}\begin{pmatrix}2k & 0\\ 23 & -5k\end{pmatrix}$ or e.g. $\begin{pmatrix}1 & 0\\ \frac{23}{2k} & \frac{-5}{2}\end{pmatrix}$	Correct matrix	A1
			(2)
			Total 4

Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(z)=2z^4-19z^3+Az^2+Bz-156$		
(a)	$(z=)5+\mathrm{i}$	Correct complex number	B1
			(1)
	Mark (b) and (c) together – ignore any labelling seen. **Award marks in the order given for their choice of method**		
(b)/(c) With (b) first	$z=5\pm\mathrm{i}\Rightarrow(z-(5+\mathrm{i}))(z-(5-\mathrm{i}))=...$ Or e.g. Sum of roots = 10 Product of roots = 26	Correct strategy to find the quadratic factor using the conjugate pair	M1
	$z^2-10z+26$	Correct quadratic	A1
	$\mathrm{f}(z)=(z^2-10z+26)(2z^2+...z+k)$	Attempts to find the other quadratic. May use inspection (apply rules for quadratic factorisation ie $"26"\lvert k\rvert=156$) or e.g. long division with quotient $2z^2+...z+...$	M1
	NB long division gives quotient $2z^2+z+(A-42)$ and remainder $(10A+B-446)z+936-26A$		
	$2z^2+z-6$	Correct quadratic	A1
	$\Rightarrow z=\frac{3}{2},\ -2\ \ (,5\pm\mathrm{i})$	Correct real roots. The complex roots do not have to be stated.	A1
			(5)
	$\mathrm{f}(z)=(z^2-10z+26)(2z^2+z-6)$ $=...$	Multiplies out both quadratics or extracts the terms needed	M1
	$A=36,\ B=86$	Correct values (can be seen in the quartic equation)	A1
			(2)
			Total 8
(b)/(c) With (c) first	952+960i-2090-24A+10Ai+5Bi-156=0	Substitute $(5+\mathrm{i})$ into the quartic (by calculator) and equate real and imag parts (can be done with $(5-\mathrm{i})$)	M1
	-1294+24A+5B=0 -446+10A+B=0	Correct equations	A1
	A=36 B=86	M1 Solve simultaneously A1 One correct A1 Both correct	M1 A1A1
			(5)
	$2z^4-19z^3+36z^2+86z-156=0$ $z=...$	Solve the equation by long division, inspection or by calculator	M1
	$\Rightarrow z=\frac{3}{2},\ -2\ \ (,5\pm\mathrm{i})$	Correct real roots. The complex roots do not have to be stated.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$2x^2 - 3x + 5 = 0$		
(a)	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{5}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right) = -\frac{11}{4} \ (= -2.75)$	Correct value Allow to come from $\alpha + \beta = -\frac{3}{2}$	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Reaches an identity ready for substitution	M1
	$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{2}\right) = -\frac{63}{8} \ (= -7.875)$	Correct value	A1
			(4)
(c)	$\text{Sum} = \alpha^3 + \beta^3 - (\alpha + \beta) = -\frac{63}{8} - \frac{3}{2}\left(= -\frac{75}{8}\right)$	Attempts sum Allow eg $(\alpha^3 - \beta) + (\beta^3 - \alpha)$ followed by $(\alpha^3 + \beta^3) + (\alpha + \beta) = \ldots$	M1
	$\text{Prod} = (\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta$ and $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$	Expands $(\alpha^3 - \beta)(\beta^3 - \alpha)$ and uses a correct identity for $\alpha^4 + \beta^4$	M1
	Alt identities: $\alpha^4 + \beta^4 =$ $(\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2$; $\alpha^4 + \beta^4 = (\alpha^3 + \beta^3)(\alpha + \beta) - \alpha\beta(\alpha^2 + \beta^2)$		
	$(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta = \left(\frac{5}{2}\right)^3 + \frac{5}{2} - \left(\left(-\frac{11}{4}\right)^2 - 2\left(\frac{5}{2}\right)^2\right) = \frac{369}{16}$		A1
	$x^2 + \frac{75}{8}x + \frac{369}{16} (= 0)$	Applies $x^2 -$ (their sum)x + their prod (= 0)	M1
	$16x^2 + 150x + 369 = 0$	Allow any integer multiple	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$x=9t^2, y=18t \Rightarrow \frac{dy}{dx}=\frac{18}{18t}$ or $y^2=36x \Rightarrow 2y\frac{dy}{dx}=36 \Rightarrow \frac{dy}{dx}=\frac{18}{y}=\frac{18}{18t}$ or $y^2=36x \Rightarrow y=6\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{3}{\sqrt{x}}=\frac{3}{3t}$	Correct $\frac{dy}{dx}$ in terms of t There must be evidence of use of calculus ($\frac{dy}{dx}=\frac{1}{t}$ with no working scores B0)	B1
	$m_T=\frac{1}{t} \Rightarrow m_N=-t$	Correct use of the perpendicular gradient rule.	M1
	$y-18t=-t\left(x-9t^2\right)$	Correct straight line method for the normal. Must use their perpendicular gradient – not dy/dx. (Any complete method – use of $y = mx + c$ requires an attempt at "c")	dM1
	$y+tx=9t^3+18t$ *	Cso All previous marks must have been earned	A1*
			(4)
(b)	$x=54, y=0 \Rightarrow 54t=9t^3+18t$ $\Rightarrow 9t^3-36t=0$	Substitutes $x = 54$ and $y = 0$ into the equation from part (a) and attempts to collect terms.	M1
	$9t^3-36t=0 \Rightarrow 9t\left(t^2-4\right)=0$ $\Rightarrow t=\pm 2 \Rightarrow y \pm 2x=9(\pm 2)^3+18(\pm 2)$	Solves to obtain at least one non zero value for t and attempts at least one normal equation	dM1
	$y=-2x+108$ **or** $y=2x-108$	One correct equation in any equivalent form	A1
	$y=-2x+108$ **and** $y=2x-108$	Both correct and in the required form	A1
			(4)
(c)	$x=-9 \Rightarrow y=18+108$ or $-18-108$	Uses $x = -9$ to find the y coordinate of A or B	M1
	Area = $\frac{1}{2}\times 252\times 18$	Fully correct strategy for the area Award M0 if their x coord of the focus is not doubled	M1
	= 2268	Cao	A1
			(3)
			Total 11
ALT	Last 2 marks by "shoelace" method: eg $\left\lvert \frac{1}{2}\begin{vmatrix} -9 & 9 & -9 & -9 \\ 126 & 0 & -126 & 126 \end{vmatrix} \right\rvert$ $=\left\lvert\frac{1}{2}\left(9\times -126-9\times 126-\left(-9\times -126+9\times 126\right)\right)\right\rvert$	Their coordinates with first and last the same ½ must be included Attempt to expand also needed	M1
	= 2268	Must be positive	A1

Question Number	Scheme	Notes	Marks
7(a)	$\mathbf{A}^2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	Rotation −60° (anticlockwise) about the origin	Rotation	M1
		−60° (anticlockwise) (Or 60° clockwise or 300° (anticlockwise)) about (0, 0)	A1
			(2)
(c)	$n = 12$	Cao but can be embedded ie $A^{12} = I$	B1
			(1)
(d)	$\mathbf{B} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(e)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Multiplies the right way round.	M1
	$\mathbf{C} = \begin{pmatrix} -2\sqrt{3} & -2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix Accept unsimplified	A1
			(2)
(f)	$\det \mathbf{C} = -2\sqrt{3} \times -\frac{\sqrt{3}}{2} - \frac{1}{2}(-2) = 4$ So area of P is $\frac{20}{\det \mathbf{C}} = \ldots$	Attempts determinant of **C** (or deduces area scale factor is 4) and divides into 20	M1
	$= 5$	Cao Must follow a correct matrix in (e)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
8	$\sum_{r=0}^{n}(r+1)(r+2)$		
(a)	$\sum_{r=0}^{n} r^2+3r+2 = 2+\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+2n$ M1: Attempt to use at least one of the standard formulae correctly A1: For $\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+(2n \text{ or } 2n+2)$ A1:Fully correct expression		M1A1A1
	$\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+2n+2=(n+1)\left[\frac{1}{6}n(2n+1)+\frac{3}{2}n+2\right]$ Attempt to factorise $(n+1)$ It is a "show" question so this must be seen (in any equivalent form). If their expression does not allow for factorising $(n+1)$ score M0		M1
	$\frac{1}{3}(n+1)\left[n^2+5n+6\right]$	May obtain a cubic and extract a different factor ie $n + 2$ or $n + 3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)$*	Cso At least one intermediate step in the working must be seen.	A1*
			(5)
(a) Way 2	$\sum_{r=0}^{n}(r+1)(r+2)=\sum_{r=1}^{n+1}r(r+1)$ $=\sum_{r=1}^{n+1}r^2+r=\frac{1}{6}(n+1)(n+2)(2(n+1)+1)+\frac{1}{2}(n+1)(n+2)$ M1: Attempt to use at least one of the standard formulae correctly with $n = n + 1$ A1: For $\frac{1}{6}(n+1)(n+2)(2(n+1)+1)$ or $\frac{1}{2}(n+1)(n+2)$ A1:Fully correct expression		M1A1A1
	$\frac{1}{6}(n+1)(n+2)(2(n+1)+1)+\frac{1}{2}(n+1)(n+2)=(n+1)\left[\frac{1}{6}(n+1)(2n+3)+\frac{1}{2}(n+2)\right]$ Attempt to factorise $(n+1)$ (see additional comments above)		M1
	$\frac{1}{3}(n+1)\left[n^2+5n+6\right]$	May obtain a cubic and extract a different factor ie $n + 2$ or $n + 3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)$*	Cso At least one intermediate step in the working must be seen.	A1*
(b)	Upper limit = 99	Correct upper limit	B1
	$10\times11+11\times12+12\times13+...+100\times101=\sum_{r=0}^{99}(r+1)(r+2)-\sum_{r=0}^{8}(r+1)(r+2)$ Fully correct strategy for the sum using their upper limit for the first sum and upper limit 8 for the second in the result from (a). Lower limits 0 or 1		M1
	$=\frac{1}{3}(100)(101)(102)-\frac{1}{3}(9)(10)(11)$ $= 343\ 070$	Correct value	A1

			(3)
			Total 8

Question Number	Scheme	Notes	Marks
9(i)	$u_n = 5\times 2^{n-1} - n\times 2^n$		
	$n=1 \Rightarrow u_1 = 5\times 2^0 - 1\times 2 = 3$ (Shows the result is true for $n = 1$)		B1
	Assume true for $n = k$ so that $u_k = 5\times 2^{k-1} - k\times 2^k$		
	$u_{k+1} = 2\left(5\times 2^{k-1} - k\times 2^k\right) - 2^{k+1}$	Attempts u_{k+1} using the recurrence relationship	M1
	$= 5\times 2^k - k\times 2^{k+1} - 2^{k+1}$	Correct expanded expression	A1
	$= 5\times 2^k - (k+1)2^{k+1}$	Achieves this result with no errors	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted.		
			(5)
(ii)	$\mathrm{f}(n) = 5^{n+2} - 4n - 9$		
	$\mathrm{f}(1) = 125 - 4 - 9 = 112 = 16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume true for $n = k$ so that $5^{k+2} - 4k - 9$ is divisible by 16		
	$\mathrm{f}(k+1) = 5^{k+3} - 4(k+1) - 9$	Attempts $\mathrm{f}(k + 1)$	M1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + \ldots$	Attempts to express in terms of $\mathrm{f}(k)$	dM1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + 16k + 32$	Correct expression for $\mathrm{f}(k + 1)$	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		**(5)**
			Total 10

ii ALT 1	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-mf(k)=5^{k+3}-4(k+1)-9-m\left(5^{k+2}-4k-9\right)$ Attempt f(k+1) – mf(k)		M1
	$=(5-m)\left(5^{k+2}-4k-9\right)+...$	Attempts to express in terms of f(k)	dM1
	$f(k+1)=5\times\left(5^{k+2}-4k-9\right)+16k+32$	Correct expression for f(k + 1)	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 2	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-f(k)=5^{k+3}-4(k+1)-9-\left(5^{k+2}-4k-9\right)$ Attempt f(k+1) – f(k)		M1
	$f(k+1)-f(k)=5\times 5^{k+2}-5^{k+2}-4k-4-9+4k+9$ $=4\times 5^{k+2}-4=4\left(5^{k+2}-1\right)$ Obtains a simplified expression for the difference **and** attempts to prove $\left(5^{k+2}-1\right)$ is divisible by 4 using induction		dM1
	Correct proof for $\left(5^{k+2}-1\right)$ being divisible by 4 and states that thus as the difference is divisible by 16, $f(k+1)$ is divisible by 16		A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1 cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 3	$f(1)=125-4-9=112=16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	f(k) is divisible by 16 so set $f(k)=16\lambda$		
	$5^{k+2}=16\lambda+4k+9$		M1
	$f(k+1)=5^{k+3}-4(k+1)-9$ $=5\times 5^{k+2}-4k-13=5(16\lambda+4k+9)-4k-13$	Expresses f(k + 1) in terms of λ and k and collects terms	dM1
	$=80\lambda+16k+32$	Correct expression May have factor of 16 taken out	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		